$$\begin{cases} y = -\frac{13}{12} \\ 4x - 2\cdot y = -1 \end{cases} \qquad \begin{cases} y = -\frac{13}{12} \\ 4\cdot x - 2\cdot\left(-\frac{13}{12}\right) = -1 \end{cases} \qquad \begin{cases} y = -\frac{13}{12} \\ x = -\frac{19}{24} \end{cases}$$

es. p. 37 n°25

$$(x+2)(1-2x) = x\cdot 2^x + \underbrace{2^{x+1}}_{1}$$

parte algebrica e parte trascendente.

$$(x+2)(1-2x) = x\cdot 2^x + 2^x\cdot 2$$

$$(x+2)(1-2x) = 2^x(x+2)$$

se $x \neq -2$
ricavo 2^x dividendo tutto per $x+2$

$$1-2x = 2^x$$

Si procede con la risoluzione grafica.

p. 36 n° 10

Esercizio p. 39 n° 20

a) $\frac{3^{x+1}}{27^{2x}} < \frac{1}{3^{x^2+5}}$ ho solo potenze di 3

$$\frac{3^{x+1}}{(3^3)^{2x}} < 3^{-x^2-5}$$

$$\frac{3^{x+1}}{3^{6x}} < 3^{-x^2-5}$$

$$3^{x+1-6x} < 3^{-x^2-5}$$

$$3^{-5x+1} < 3^{-x^2-5}$$

si ha una diseq. esponenziale base > 1

la diseq esponenz. è equivalente ad una disequazione DELLO STESSO VERSO tra gli esponenti.

ha le stesse soluzioni

$$-5x+1 < -x^2-5$$

$$x^2-5x+6<0$$

Risolvo l'eq. associata alla diseq:

$x=3$, $x=2$

la diseq. è risolta per valori interni.

$\boxed{2<x<3}$ $S=(2,3)$

b) $4\sqrt{2} < \frac{1}{\sqrt{8^x}}$ ho solo potenze di 2

$2 \cdot 2^{\frac{1}{2}} < \frac{1}{\sqrt{2^{3x}}}$ $\longrightarrow$ $2^{1+\frac{1}{2}} < \frac{1}{2^{+\frac{3x}{2}}}$

$2^{\frac{5}{2}} < 2^{-\frac{3}{2}x}$ base > 1

$$\frac{5}{2} < -\frac{3}{2}x$$

a parità di base vale la stessa relaz. tra gli esponenti.

$$2 \cdot \frac{5}{2} < -\frac{3}{2}x \cdot 2$$

$$5 < -3x$$

$$-5 > 3x$$

$$-\frac{5}{3} > x$$

$$\boxed{x < -\frac{5}{3}} \qquad S = \left(-\infty, -\frac{5}{3}\right)$$

Esercizio p. 39 n° 20 c

$$\sqrt{2^x} \geq 8 \cdot \sqrt[3]{4^{x-1}}$$

ho solo potenze di 2

$$2^{\frac{x}{2}} \geq 2^3 \cdot \sqrt[3]{(2^2)^{x-1}}$$

$$2^{\frac{x}{2}} \geq 2^3 \cdot 2^{\frac{2x-2}{3}}$$

$$2^{\frac{x}{2}} \geq 2^{3 + \frac{2x-2}{3}}$$

$$\frac{x}{2} \geq 3 + \frac{2x-2}{3}$$

$$\frac{x}{2} \geq \frac{9+2x-2}{3}$$

$$\frac{x}{2} \geq \frac{7}{3} + \frac{2}{3}x$$

$$\left(\frac{1}{2} - \frac{2}{3}\right)x \geq \frac{7}{3}$$

$$\frac{3-4}{6}x \geq \frac{7}{3} \qquad -\frac{x}{6} \geq \frac{7}{3} \qquad \frac{x}{2} \leq -7$$

$$\boxed{x \leq -14}$$

esercc. p. 39 n° 30

$4^x - 3 \cdot 2^x + 2 > 0$

$2^{2x} - 3 \cdot 2^x + 2 > 0$ porre $\begin{cases} 2^x = y \\ 4^x = y^2 \end{cases}$

$y^2 - 3y + 2 > 0$ (1)

$y = 2$
$y = 1$ soluz. dell' eq. associata

La diseq. (1) è risolta per valori esterni.

$y < 1 \vee y > 2$ ma $2^x = y$

$2^x < 1 \vee 2^x > 2$

$2^x < 2^0 \vee 2^x > 2^1$

$$\boxed{x < 0 \vee x > 1}$$

eserc. p. 39 n° 34

a) $\frac{1}{3} < 9^{2x-1} < 27$ doppia disequazione.

equivale ad un sistema di disequaz.

$$\begin{cases} \frac{1}{3} < 9^{2x-1} \\ 9^{2x-1} < 27 \end{cases}$$

$3^{-1} < (3^2)^{2x-1} < 3^3$

$3^{-1} < 3^{4x-2} < 3^3$

$-1 < 4x - 2 < 3$ Devo ricavare l' incognita sommo $+2$

$1 < 4x < 5$

$\frac{1}{4} < x < \frac{5}{4}$

p.39 n°34 b

$5^x - 4 \geq 5^{1-x}$ ho 3 termini; in uno non compare l'incognita

$5^x - 4 - \boxed{5 \cdot 5^{-x}} \geq 0$

Sia $y = 5^x$

$y - 4 - 5\frac{1}{y} \geq 0$ diseq. frazionaria

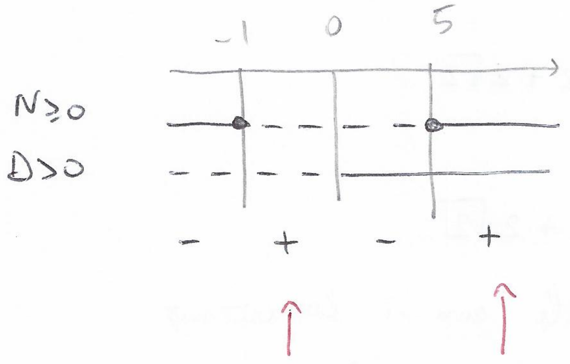
$$\frac{y^2 - 4y - 5}{y} \geq 0$$

$N \geq 0$ $y^2 - 4y - 5 \geq 0$

$\left.\begin{array}{l} y = 5 \\ y = -1 \end{array}\right\}$ soluzioni dell'eq. di 2° gr. associata

$y \leq -1 \vee y \geq 5$

$D > 0$ $y > 0$



$-1 \leq y < 0 \vee y \geq 5$ ma $y = 5^x$

$-1 \leq 5^x < 0 \vee 5^x \geq 5$

imposs. $\vee$ $x \geq 1$

$$\boxed{x \geq 1}$$

$S = [1, +\infty)$

esempio 1

$$2^x - \frac{4}{2^x} \geq -4 \qquad y = 2^x$$

$$y - \frac{4}{y} + 4 \geq 0$$ diseq. frazionaria.

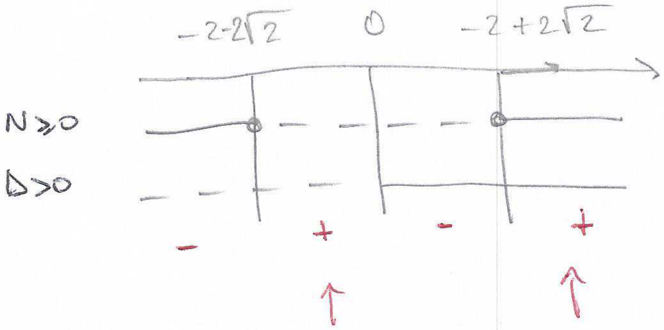
$$\frac{y^2 - 4 + 4y}{y} \geq 0$$

$N \geq 0 \quad y^2 + 4y - 4 \geq 0 \qquad \frac{\Delta}{4} = \left(\frac{b}{2}\right)^2 - a \cdot c = 4 + 4 = 8$

$$y_{1/2} = -2 \pm \sqrt{8} = -2 \pm 2\sqrt{2} = \begin{cases} -2 - 2\sqrt{2} \hat{=} -4{,}828 \\ -2 + 2\sqrt{2} \hat{=} 0{,}828 \end{cases}$$

$y \leq -2 - 2\sqrt{2} \quad \vee \quad y \geq -2 + 2\sqrt{2}$

$D > 0 \quad y > 0$



$-2 - 2\sqrt{2} \leq y < 0 \quad \vee \quad y \geq -2 + 2\sqrt{2}$

$-2 - 2\sqrt{2} \leq 2^x < 0 \quad \vee \quad 2^x \geq -2 + 2\sqrt{2}$

impossibile. risolvibile con i logaritmi.

$$x \geq \log_2 (2\sqrt{2} - 2)$$

esempio 2

$$2^x + \frac{4}{2^x} > 4$$

$$y = 2^x$$

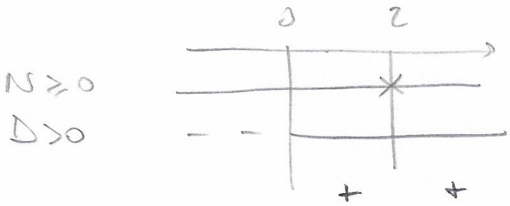
$$y + \frac{4}{y} - 4 > 0$$

$$\frac{y^2 + 4 - 4y}{y} > 0$$

$N \geq 0 \qquad y^2 - 4y + 4 > 0$

$(y-2)^2 > 0 \quad \forall y \quad$ con $\quad y \neq 2$

$D > 0 \qquad y > 0$



$0 < y < 2 \quad \vee \quad 2 < y < +\infty$

$\underbrace{0 < 2^x}_{\text{sempre}} < 2 \quad \vee \quad 2 < 2^x$

$2^x < 2 \qquad 1 < x$

$x < 1 \quad \vee \quad x > 1$

$$S = (-\infty, 1) \cup (1, +\infty)$$

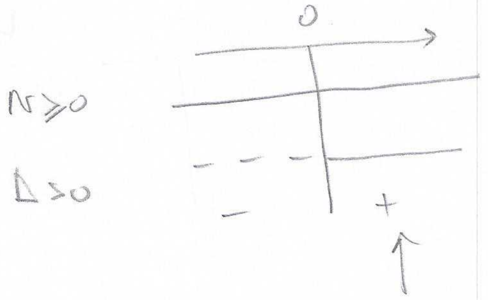
esercizio p. 39 n° 34 e

$$2^x + \frac{4}{2^x} - 4 \geq 0 \qquad y = 2^x$$

$$y + \frac{4}{y} - 4 \geq 0 \longrightarrow \frac{y^2 - 4y + 4}{y} \geq 0$$

$N \geq 0 \quad (y-2)^2 \geq 0 \quad \forall y$

$D > 0 \quad y > 0$



$y > 0$

$2^x > 0 \quad \underbrace{\forall x \in \mathbb{R}} \qquad S = \mathbref{R}$

eserc. p. 40 n° 48

$3^{x-2} : 9^{x-2} < 27^x$

$3^{x-2} : (3^2)^{x-2} < (3^3)^x$

$3^{x-2} : 3^{2x-4} < 3^{3x}$

$3^{x-2-2x+4} < 3^{3x}$

$3^{-x+2} < 3^{3x}$

$-x+2 < 3x$

$-4x < -2$

$-2x < -1$

$2x > 1$

$x > \frac{1}{2}$

esercizio p.40 n° 49

$9^x - 6 \cdot 3^x - 27 < 0$

$3^{2x} - 6 \cdot 3^x - 27 < 0 \qquad y = 3^x$

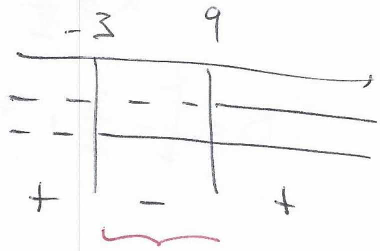
$y^2 - 6y - 27 < 0$

$y = 9 \qquad y = -3$

$(y-9)(y+3) < 0$

$1° \ f > 0 \qquad y - 9 > 0 \rightarrow y > 9$

$2° \ f > 0 \qquad y > -3$



soluz

$-3 < y < 9$

$\underline{-3 <} \, 3^x < 3^2$

doppia disequazione: equivale ad un sistema di disequaz.

$$\begin{cases} 3^x > -3 & \forall x \\ 3^x < 3^2 & \rightarrow \quad x < 2 \end{cases}$$

Soluz: $\boxed{x < 2}$

$S = (-\infty, 2)$

eserc. p.41 n° 52

$\frac{2}{5} < \left(\frac{2}{5}\right)^x < \frac{25}{4}$ mi riconduco alla stessa base

$\left(\frac{2}{5}\right)^1 < \left(\frac{2}{5}\right)^x < \left(\frac{2}{5}\right)^{-2}$ base < 1

$1 > x > -2 \rightarrow -2 < x < 1$

$y - 4 \geq 0$

$y \geq 4$

$2^x \geq 2^2$

NON ESISTE

ESISTE

$\boxed{x \geq 2}$ $D = [2, +\infty)$